Signal-to-Noise Ratio Calculation for Fiber Optics Links

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In this article, the signal-to-noise ratio (SNR) effect upon the maximum transmission length of a fiberoptic system is discussed. The relationships of different system parameters are discussed. A general formula to obtain the SNR of a single mode fiberoptic system is derived.

Using the specification of a commercial avalanche photo decode, the SNR is estimated to be 5×10^{12} $m^2/\Delta\nu$ *for* $L \leqslant 30$ *km and*

$$1.1 \times \frac{10^{19}\, m^2 10^{-\alpha L/5}}{\Delta\nu}$$

for $L > 30$ *km. The main bandwidth limitating factor in multimode fiberoptic systems is found to be intermodal dispersion.*

It is concluded that for frequency standard transmission a respectable SNR can be obtained for different fiber optic systems, providing that we are operating within the bandwidth of the fiber.

I. Introduction

In this article, we would like to answer the following question: Given a certain signal-to-noise ratio (SNR), what is the maximum transmission length of an optical fiber system used primarily for frequency and time synchronization? In Section II, we describe the system under consideration. The SNR for single mode optical fiber systems is derived in Section III and estimated numerically in Section IV. In Section V, the SNR for multi-mode fiber links is derived and compared with single-mode fiber links.

II. The System

Let us consider the fiber optics link sketched in Fig. 1. The laser diode oscillates at a single optical frequency ω, and the light output is modulated by the signal source, which is a high precision frequency standard. The optical fiber has a loss of an α dB/km, is L km long, and we assume that the signal baseband lies well within the bandwidth of the fiber (which can be as high as hundreds of GHz-km for single mode fibers). The detector used is assumed to be an avalanche photodiode (APD) followed by a phase-locked amplifier. The output is delivered into a 50 Ω load. We further assume that the baseband signal is within the detection bandwidth of the APD.

We assume that the modulated light output from the laser diode is purely sinusoidal at baseband frequency Ω, with a modulation depth m. The optical power fed into the fiber is then

$$P_{\text{in}}(t) = P(1 + m \cos \Omega t) + S(t) \tag{1}$$

where P is the average optical power. $S(t)$ is the noise generated by the laser. The magnitude and spectral characteristic of this noise are quite complicated, and have been investigated by several authors (Refs. 1 and 2). It has the properties of a shot-noise-driven resonant tuned circuit. Figure 2 illustrates the spectral properties of the laser noise at different average output optical powers, P. Above lasing threshold, the output power abruptly increases while the relative noise power $\langle S^2 \rangle / P$ abruptly decreases. ($\langle \ \rangle$ denotes time average of a quantity.) Figure 3 illustrates the relative noise power density at low frequencies (< 1 GHz) vs the average output power P. However, as observed from Fig. 2, the noise power density shows a spike in the frequency range around 2-4 GHz, which can be orders of magnitude higher than in the "midband" range (~0-1 GHz). This noise spike is related to the relaxation oscillation of the laser diode. In addition to the unusually intense noise in this region, there are other undesirable effects, such as high harmonic distortion. Therefore, it is recommended that our baseband signal be below the frequency at which the noise spike appears in the laser diode response, and we shall confine our analysis similarly.

Propagation of the signal in Eq. (1) through a fiber of length L attenuates its power by a factor of $10^{-\alpha L/10}$, and hence the output optical power is

$$P_{\text{out}}(t) = 10^{-\alpha L/10} P_{\text{in}}(t) \tag{2}$$

The current generated by the APD is proportional to the incident optical power (lumping splicing and insertion losses into the attenuation term) and is given by

$$i(t) = \frac{Ge\eta}{\hbar\omega} P_{\text{out}}(t) \tag{3}$$

where G and η are respectively the avalanche gain and the internal quantum efficiency of the APD, $\hbar$ is the Planck's constant, ω is the optical frequency, and e is the electronic charge. Using Eq. (2) we have

$$\begin{aligned} i(t) = {} & \frac{GPe\eta}{\hbar\omega}(1 + m \cos \Omega t) 10^{-\alpha L/10} \\ & + \frac{GeP_B\eta}{\hbar\omega} + i_d \\ & + \frac{Ge\eta}{\hbar\omega} S(t) 10^{-\alpha L/10} \end{aligned} \tag{4}$$

where the dark current of the APD (i_d) and the background light entering the APD P_B is included. This current is then amplified by the phase-lock amplifier. We assume the phase-lock amplifier has bandwidth $\Delta\nu$ and has gain G_A.

III. Derivation of the SNR

The noise in the detection system arises from four sources: the dark current and laser source noise as in Eq. (4), plus shot noise and thermal noise (Ref. 3).

(1) The shot noise current has a mean square value of

$$\overline{i^2_{N_{\text{shot}}}} = 2e \bar{I} \Delta\nu \tag{5}$$

where $\bar{I}$ is the average amplified current from Eq. (4),

$$\bar{I} = \frac{G_A Ge\eta}{\hbar\omega} (P \times 10^{-\alpha L/10} + P_B) + G_A i_d \tag{6}$$

(2) The thermal noise current has a mean square value given by

$$\overline{i^2_{N_{\text{thermal}}}} = G_A^2 \frac{kT}{R_L} \Delta\nu \tag{7}$$

where R_L is the load resistance = 50 Ω, k is the Boltzmann constant, G_A is the amplifier current gain, and T is the temperature of the system (including the equivalent noise temperature of the amplifier).

(3) The laser source noise $S(t)$ is attenuated in passing through the fiber, and generates a mean square noise current in the APD equal to

$$\overline{i^2_{N_{\text{laser}}}} = \left(\frac{Ge\eta}{\hbar\omega} \right)^2 \langle S^2 \rangle \, 10^{-\alpha L/5} \Delta\nu \tag{8}$$

The total noise power is thus

$$\overline{i^2_{N_{\text{shot}}}} + \overline{i^2_{N_{\text{thermal}}}} + \overline{i^2_{N_{\text{laser}}}} =$$

$$2eG_A \left\{ \frac{Ge\eta}{\hbar\omega} (P \times 10^{-\alpha L/10} + P_B) + i_d \right\} \Delta\nu \tag{9}$$

$$+ G_A^2 \frac{kT}{R_L} \Delta\nu + \left(\frac{GG_A \eta}{\hbar\omega}\right)^2 \langle S^2 \rangle \, 10^{-\alpha L/5} \, \Delta\nu$$

The signal power, from Eq. (4), is:

$$\overline{i_S^2} = \frac{1}{2} \left(\frac{G_A \, GPe\eta}{\hbar\omega} m \right)^2 10^{-\alpha L/5} \tag{10}$$

The signal-to-noise ratio is then:

$$\frac{\text{S}}{\text{N}} = \frac{\frac{1}{2} \left(\frac{G_A \, GPe\eta}{\hbar\omega} m \right)^2 10^{-\alpha L/5}}{2eG_A \frac{Ge\eta}{\hbar\omega} (P \times 10^{-\alpha L/10} + P_B) \, \Delta\nu + 2e \, G_A \, i_d \, \Delta\nu + \frac{kTG_A^2}{R_L} \Delta\nu + \left(\frac{GG_A \eta}{\hbar\omega}\right)^2 \langle S^2 \rangle \, 10^{-\alpha L/5} \, \Delta\nu} \tag{11}$$

IV. Numerical Estimations

We shall estimate the contributions of the four terms in the denominator of Eq. (11), which are due to, respectively, shot, dark current, and thermal and laser noise.

Typical parameters for a silicon avalanche photodiode (RCA model C30902E) are:

$\frac{eG\eta}{\hbar\omega}$ = 65 amps/watt around the wavelength of GaAs laser diodes,

i_d = dark current = 16 nA

Typically, the power coupled from the laser diode into the optical fiber is approximately 1 mW. Assuming the background leakage light P_B to be zero, the shot noise contribution is

$$2e \frac{G_A Ge\eta}{\hbar\omega} P \times 10^{-\alpha L/10} \approx 2 \times 10^{-20} \, G_A \times 10^{-\alpha L/10} \, \text{amp}^2/\text{Hz}$$

The dark current contribution is

$$2ei_d \, G_A \approx 5.1 \times 10^{-27} \, G_A \, \text{amp}^2/\text{Hz}$$

Assuming an ambient temperature of 300 K, and the noise figure of the amplifier to be 2 dB, the thermal noise contribution is

$$G_A^2 \frac{kT}{R_L} \simeq 1.31 \times 10^{-22} \, G_A^2 \, \text{amp}^2/\text{Hz}$$

Lastly, from Figs. 2 and 3, the laser noise power is, assuming an average optical output of 3 mW from the laser diode, approximately $\langle S^2 \rangle = 10^{-19}$ watt/Hz. The noise current at the detector due to laser noise is

$$\left(\frac{GG_A \eta}{\hbar\omega}\right)^2 \langle S^2 \rangle \, 10^{-\alpha L/5} \simeq$$

$$4.2 \times 10^{-16} \times 10^{-\alpha L/5} \, G_A^2 \, \text{amp}^2/\text{Hz}$$

The signal power is

$$\frac{1}{2} \left(\frac{Ge\eta}{\hbar\omega} Pm \right)^2 10^{-\alpha L/5} G_A^2 \simeq$$

$$2.1 \times 10^{-3} \, m^2 \, 10^{-\alpha L/5} \, G_A^2 \, \text{amp}^2$$

Thus, the signal-to-noise ratio is

$$\frac{\text{S}}{\text{N}} = \frac{A \, m^2 10^{-\alpha L/5} \, G_A}{(B \, 10^{-\alpha L/10} + C + G_A D + G_A E \times 10^{-\alpha L/5}) \, \Delta\nu} \tag{12}$$

where

$A = 2.1 \times 10^{-3}$ (signal)

$B = 2 \times 10^{-20}$ (shot noise)

$C = 5.1 \times 10^{-27}$ (dark current noise)

$D = 1.31 \times 10^{-22}$ (thermal noise)

$E = 4.2 \times 10^{-15}$ (laser noise)

Equation (12) is our principal result. In Fig. 4, we plot the contribution of the various noises vs the link length-attenuation product. If the attenuation is 1 dB/km, then the x-axis will be scaled in km. The amplifier gain G_A is assumed to be 100, and for a fiber loss of 1 dB/km, we see that laser noise dominates for $L \leqslant 30$ km, and that the signal-to-noise ratio is approximately constant.

$$\frac{S}{N} \simeq \frac{A\, m^2}{E\, \Delta\nu} \simeq 5 \times 10^{12} \frac{m^2}{\Delta\nu} \tag{13}$$

Beyond 30 km, we have

$$\frac{S}{N} \simeq \left(\frac{A}{D}\right) \frac{m^2\; 10^{-\alpha L/5}}{\Delta\nu} \simeq 1.1 \times 10^{19} \frac{m^2\; 10^{-\alpha L/5}}{\Delta\nu} \tag{14}$$

In Fig. 5, we plot SNR vs αL for $\Delta\nu = 10$ Hz. Thus, a respectable SNR can be maintained for a link length as long as 70 km with fibers of about 1 dB/km loss.

Since the bandwidth $\Delta\nu$ of the amplifier appears in the denominator of SNR, it follows that for wideband signal transmission of bandwidth B the SNR will be that shown in Fig. 5 minus $10 \log(B/10)$ dB (since we have assumed $\Delta\nu = 10$ Hz in Fig. 5).

V. SNR Calculation for Multimode Fibers

Our previous calculations have assumed that the baseband signal falls within the transmission bandwidth of the optical fiber. For single-mode fibers, the bandwidth-length product can be as high as 100 GHz-km (assuming a single mode laser source) while multimode fibers are commonly in the range of 1 GHz-km. For a 100 km link with a baseband signal of 500 MHz, say, the signal lies comfortably within the bandwidth of single mode fibers but not multimode fibers. In the following we include this effect and examine its consequences.

The bandwidth-limiting factor in multimode fibers is intermodal dispersion. Multimode fibers support a large number (several hundred) of *transverse* optical modes due to their relatively large size. These transverse modes, however, each carrying a sinusoidal baseband modulation, travel down the fiber at different group velocities. Hence, the received signal is a sum of sinusoids of different phases. The result is a lowering of the modulation depth as compared with that at the input (where each mode carries modulation with the same phase).

The amount of intermodal dispersion in a multimode fiber is usually specified in time-spread/km. When an optical impulse is fed into the fiber, the output can be approximated by a gaussian pulse with half-width increasing linearly with fiber length. (With strong mode-coupling and for sufficiently long fiber length, the gaussian width increases as the square root of distance instead of linearly – see Ref. 4.) A typical number for multimode fiber is approximately 1 ns/km. Thus, the impulse response of the fiber is

$$g(t) = e^{-t^2/L^2}/L\sqrt{\pi} \tag{15}$$

where t is measured in ns, and L is the total length of the fiber in km. The factor $L\sqrt{\pi}$ is required for power conservation.

The response of the fiber due to an arbitrary input is given by the convolution integral

$$P_{\text{out}}(t) = \int_{-\infty}^{\infty} P_{\text{in}}\,(t-\tau)\, g(\tau)\, d\tau \tag{16}$$

For a purely sinusoidal input with modulation depth m,

$$P_{\text{in}}(t) = P(1 + m \cos\, \Omega t)$$

and the output is

$$\begin{aligned} P_{\text{out}}(t) &= \frac{P}{L\sqrt{\pi}} \int_{-\infty}^{\infty} (1 + m \cos \Omega\,(t-\tau)) e^{-\tau^2/L^2}\, d\tau \\ &= P + \frac{mP}{L\sqrt{\pi}} Re \left(e^{i\Omega t} \int_{-\infty}^{\infty} e^{-i\Omega t} e^{-\tau^2/L^2}\, d\tau \right) \\ &= P + \frac{mP}{L\sqrt{\pi}} Re \left(e^{i\Omega t} e^{-\left(\frac{L\Omega}{2}\right)^2} \int_{-\infty}^{\infty} e^{-\left(\frac{\tau}{L} - \frac{iL\Omega}{2}\right)^2} d\tau \right) \\ &= P \left(1 + m e^{-\left(\frac{L\Omega}{2}\right)^2} \cos \Omega\, t \right) \end{aligned} \tag{17}$$

Hence the modulation depth is reduced to an effective value

$$m' = me^{-(\Omega L/2)^2} \tag{18}$$

Since, according to Eq. (12), the signal-to-noise ratio is proportional to m^2, it follows that SNR decreases as $\exp(-(\Omega_L)^2/2)$. In Fig. 6, we plot SNR vs αL for various baseband modulation frequencies. The effect is drastic once the baseband falls outside the fiber bandwidth. As long as this is not the case, the difference between single mode and multimode fibers is minimal. For a 100 km link, this requirement is satisfied if the baseband frequency is less than several GHz for single mode fibers and ~10 MHz for multimode fibers.

VI. Conclusion

The SNR attainable with single mode and multimode fiber optics links was calculated from fundamental noise considerations. It was found that for single mode fibers, laser noise dominates the noise contributions for links less than 30 km long, while thermal noise dominates for longer links. Multimode fibers degrade SNR for long links because of intermode dispersion. For frequency standard transmission, as long as the baseband modulation signals are within the bandwidth of the fibers, respectable SNR can be attained with low-loss fibers (~1 dB/km) for links as long as 70 km. For wideband transmission SNR is decreased by a factor equal to the ratio of the bandwidth.

References

1. D. E. McCumber, "Intensity fluctuations in the output of CW laser oscillators. I," *Physical Review*, Vol. 141, No. 1, p. 306, 1966.
2. D. J. Morgan and M. J. Adams, "Quantum noise in semiconductor lasers," *Phys. Stat. Sol.* (a), 11, p. 243, 1972.
3. A. Yariv, *Introduction to optical electronics*, Holt, Rinehart and Winston, 1971.
4. S. D. Personik, "Time dispersion in dielectric waveguides," Bell System Technical Journal, Vol. 50, p. 843, 1971.

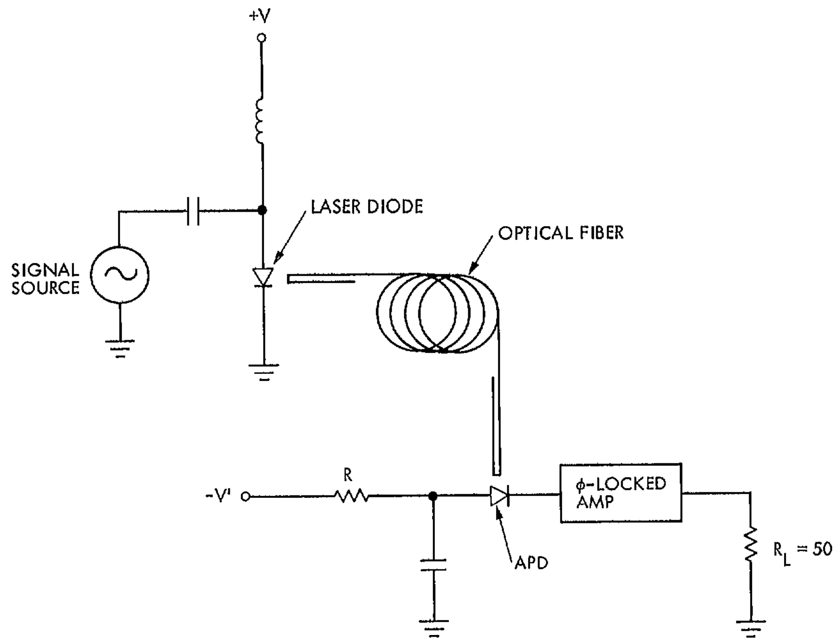
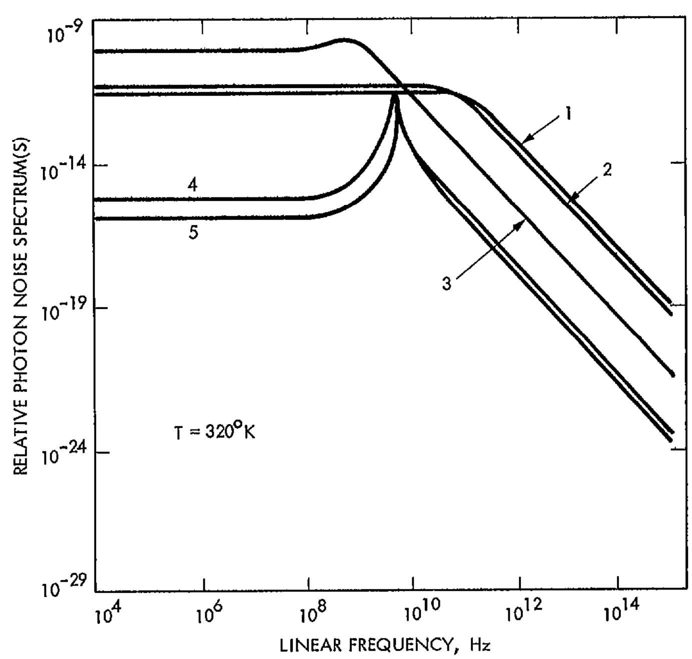


Fig. 1. Schematic fiber optics link



1. AVERAGE LASER OUTPUT POWER = 7.88×10^{-8} WATT
2. 1.98×10^{-7} WATT
3. 2.39×10^{-5} WATT
4. 2.38×10^{-3} WATT
5. 4.76×10^{-3} WATT

Fig. 2. Relative laser noise spectrum for various laser power outputs (from Ref. 2, assuming lasing wavelength = 9000 Å, laser cavity lifetime = 5 ps)

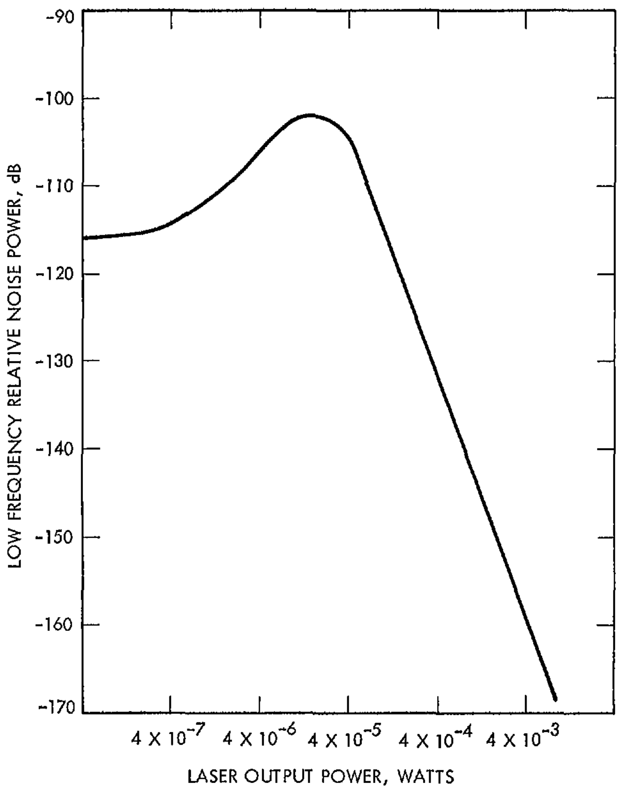


Fig. 3. Low frequency (<1 GHz) relative noise power vs average laser output power

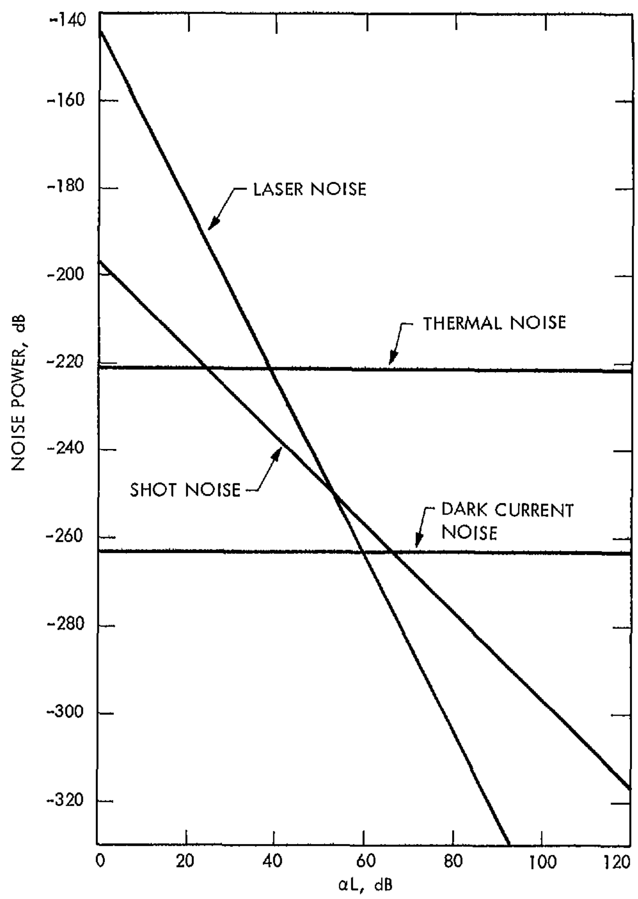


Fig. 4. Relative contributions of various noises vs total attenuation

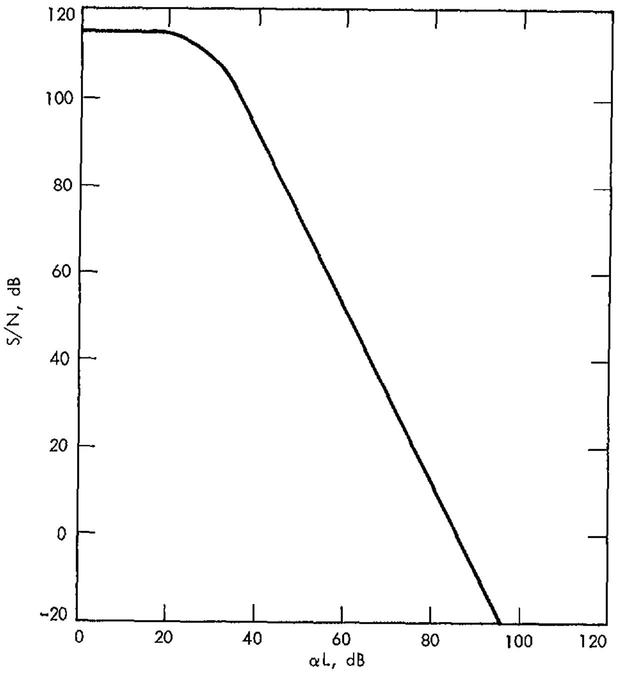


Fig. 5. Signal-to-noise ratio vs total fiber attenuation αL

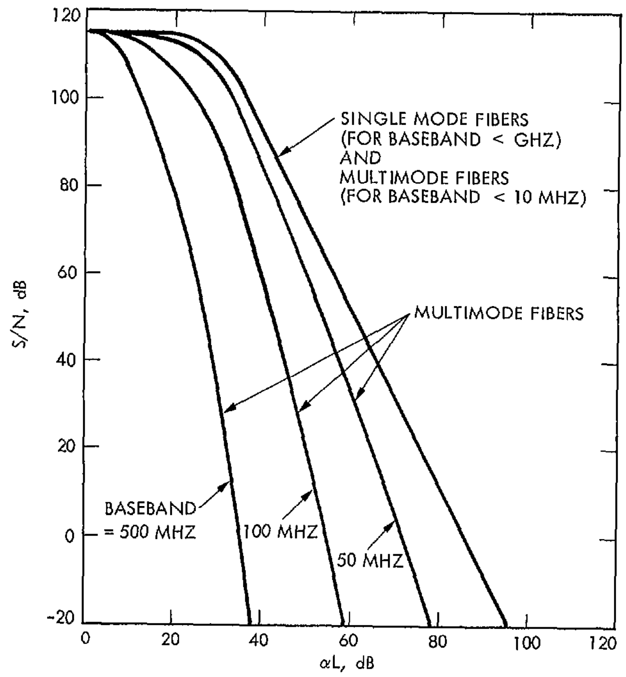


Fig. 6. Signal-to-noise ratio vs total fiber attenuation for multimode fibers